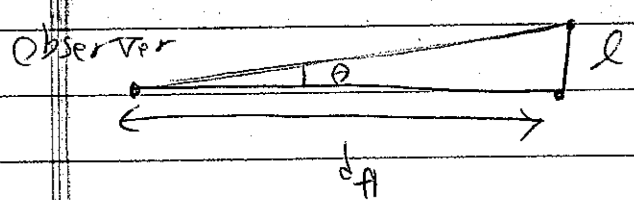
Lec 7: 02/09/2010

Observational evidence for expansion (Cont'd):

(2) Angular distance - redshift relationship. Consider an object whose size is known (a "standard ruler", e.g. a spiral galaxy). The angular distance d_A of the object from an observer is defined as:



$$d_A = \frac{l}{\theta}$$

where θ is the angular size. Note that in a static universe angular distance d_A is the same as the luminosity distance d_L, both of which are equal to the physical distance between the object and the observer.

Now consider an expanding universe with the observer at the origin ($r=0$) and the object at radial coordinate r. We then have:

$\ell = a_{(t)} r \theta \Rightarrow d_A = a_{(t)} r$

with "t" being the time that light left the object. Again, for nearby object r is small, and hence the geometry of the universe won't make a difference.

From the previous lecture, we recall that:

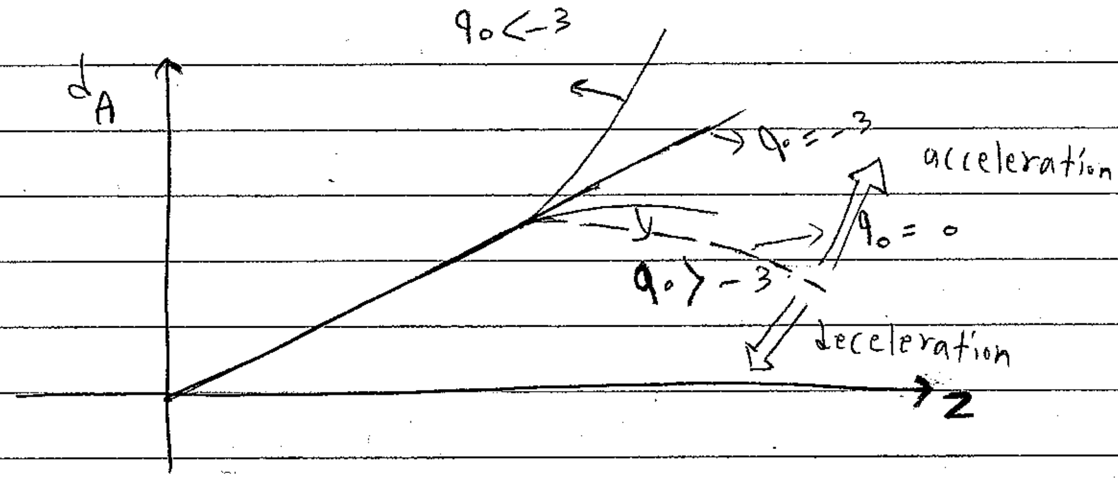
$$r \simeq \frac{H_0^{-1}}{a_{(t_0)}} \left[z - \frac{1}{2}(1+q_0) z^2 \right]$$

where subscript "0" denotes quantities at the present time. Thus:

$$d_A \simeq \underbrace{\frac{a_{(t)}}{a_{(t_0)}}}_{1+z} H_0^{-1} \left[z - \frac{1}{2}(1+q_0) z^2 \right] = \frac{H_0^{-1}}{1+z} \left[z - \frac{1}{2}(1+q_0) z^2 \right]$$

$$\simeq H_0^{-1} \left[z - \frac{1}{2}(3+q_0) z^2 \right] \Rightarrow \boxed{H_0 d_A \simeq z \left[1 - \frac{1}{2}(3+q_0) z \right]}$$

If we plot d_A vs z, we have a straight line with slope H_0^{-1} at small z, while at larger z the quadratic term becomes important.



Above the dotted curve we have accelerated expansion ($q_0 < 0$) and under that we have a decelerating expansion ($q_0 > 0$).

(3) Galaxy count - redshift relationship. Finally, we can find the expansion rate and deceleration paramete[r] at the present time by counting galaxies as a function of redshift.

Lets assume the number density of galaxies at a given radial coordinate r (the observer being at the origin) is $n_{(r)}$. Note that r is related to redshift

z, and hence we can use z instead of r.

The number of galaxies between r and $r+dr$ per solid angle $d\Omega$ is:

$$dN_{gal} = n_{(z)} dV = n_{(z)} a^3_{(t)} r^2 dr \, d\Omega$$

Again we look at small z, so the geometry of the Universe won't make a difference ($r \simeq \sin r \simeq \sinh r$). Using the expression:

$$r \simeq \frac{H_0^{-1}}{a_{(t)}} \left[z - \frac{1}{2}(1+q_0) z^2\right]$$

we find:

$$dN_{gal} \simeq a^3_{(t)} \frac{H_0^{-3}}{a^3_{(t_0)}} n_{(z)} z^2 \left[1 - 2(1+q_0) z\right] dz \, d\Omega$$

$$\Rightarrow \frac{1}{z^2} \frac{dN_{gal}}{dz \, d\Omega} \simeq \frac{n_{(z)} a^3_{(t)}}{H_0^3 a^3_{(t_0)}} \left[1 - 2(1+q_0) z\right]$$

If no glaxies are created or destroyed between redshift z and 0 (present time), the comoving number density of galaxies remains constant:

$$n_{(z)}\, a^3_{(t)} = n_0\, a^3_{(t_0)}$$

This results in;

$$\boxed{\frac{1}{z^2}\frac{dN_{gal}}{dz\, d\Omega} = \frac{n_0}{H_0^3}\left[1-2(1+q_0)z\right]}$$

From this expression we can find H_0 and q_0 after measuring N_{gal} as a function of z.

The best measurement of H_0 comes from the Hubble Space Telescope (HST). A recent measurement gives;

$$H_0 = 74.2 \pm 3.6 \frac{km}{Mpc \cdot s}$$

An earlier measurement by HST in 2001 gave:

$$H_0 = 72 \pm 8 \frac{km}{Mpc \cdot s}$$

For a matter-dominated universe we saw that $H = \frac{2}{3t}$.

Therefore, in the absence of any cosmological constant, we would have $H_0 = \frac{2}{\tau}$, with τ being the age of the universe. This was the picture before 1998 when supernova data indicated accelerated expansion as of late.

We can find a lower bound on T by looking at the oldest objects in the universe. Globular clusters are one example, very old and metal poor stars that are in clusters around the center of galaxies. Another method is to measure the abundance of heavy elements (such as Th and U) and their ratios. These elements are produced in processes during supernova explosions and undergo radioactive decay between then and now. Knowing their predicted abundances at the time of production, we can find the time between then and now after measuring their current abundances. This can then be used to set a lower bound on the lifetime of the universe.

These measurements indicate that the universe is older than 10 Gyr (current value is $\tau \sim 13.6$ Gyr). This is in contradiction with a matter-dominated universe with $H_0 \sim 70 \frac{km}{Mpc.s}$. Using $H_0 \sim 70 \frac{km}{Mpc.s}$ we find;

$$\tau = \frac{2}{3H_0} < 10 \text{ Gyr}$$

which is less than the age of globular clusters. This is known as the "age crisis" for a universe dominated by matter at the present time. It may be considered as a hint for the existence of a cosmological constant that has recently dominated the universe.